Estimating Line-Flow Limits

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Specifying reasonable line-flow limits requires two quantities: the surge impedance loading for the line and an estimate of the line length. We approximate these quantities using power flow data and assumptions of line geometry and material properties.

Note that there is not enough information to set the flow limits for transformers. This is unfortunate because transformers can be limiting transmission facilities in some systems. We identify transformers by different nominal voltage magnitudes at line terminals. Also note that this analysis does not consider security constraints (i.e., the line-flow limits derived here may not provide $N-1$ security).

1 Surge Impedance Loading

We first calculate the surge impedance loading (SIL) for the transmission line. If there is a non-zero shunt susceptance b specified for the line, calculate the characteristic impedance Z_c and the SIL from the series resistance R, series reactance X, shunt susceptance b, and the base impedance Z_{base}.

$$Z_{base} = \frac{V_{base}^2}{S_{base}} \quad [\Omega] \tag{1}$$

$$L = \frac{X_{pu}}{2\pi 60} Z_{base} \quad [\mathrm{H}] \tag{2}$$

$$R = R_{pu} Z_{base} \quad [\Omega] \tag{3}$$

$$C = \frac{B_{pu}}{2\pi 60} \frac{1}{Z_{base}} \quad [\mathrm{F}] \tag{4}$$

$$Z_c = \sqrt{\frac{R + j2\pi 60 L}{j2\pi 60 C}} \quad [\Omega] \tag{5}$$

$$SIL = \frac{V_{rated}^2}{|Z_c|} \quad [\mathrm{W}] \tag{6}$$

V_{rated} (kV)	Z_c (Ω)	SIL (MW)
69	366–400	12–13
138	366–405	47–52
230	365–395	134–145
345	280–366	325–425
500	233–294	850–1075
765	254–266	2200–2300

Table 1: Typical Characteristic Impedance and SIL Values (Table 5.2 in [2])

V_{rated} (kV)	Number of Conductors	Conductor Size (kcmil)	SIL (MW)
138	1	795	50.5
230	1	954	132
345	2	954	390
500	3	954	910
765	4	954	2210

Table 2: Typical Characteristic Impedance and SIL Values (Figure 7 in [3])

If the calculated value of SIL is within the range specified in Table 5.2 of [2] (reproduced as Table 1), or perhaps within some tolerance of this range, then use the calculated SIL value. Otherwise, there are a few possibilities:

- The line is an underground cable, which typically have low values of Z_c and correspondingly high values of SIL. Calculating the line-flow limit with the proposed method is not valid. However, the power flow data typically doesn't indicate if a line is above ground or underground so this is difficult to determine.
- The line is part of an equivalenced system. An equivalenced line does not physically exist in the system, and the specified line parameters may not make physical sense (that is, the line parameters are chosen to match the behavior of some larger system).
- The electrical characteristics of transmission facilities are being combined with the line model. For instance, the impedance of a transformer at an end of the line may be combined with the line impedance, with the transformer itself not explicitly modeled.
- The transmission element represents a double-circuit line (i.e., multiple lines on the same transmission towers). The equations used in this writeup to derive the line-flow limits do not consider the effects of double-circuit lines.
- The line parameters are in error, inconsistent, or we are overlooking something.

If b is set to zero or the value of SIL is outside the range specified in Table 5.2 [2] due to one of the four possibilities specified above, we cannot determine the value of SIL directly from the line parameter data. We can instead use the typical SIL values given in Figure 7 of [3] (reproduced as Table 2).

2 Line Length

After determining the SIL, we need to estimate the line length. Neither line lengths nor convenient proxies (e.g., the latitudes and longitudes of bus locations) are specified in typical power flow data sets. We therefore need to make some reasonable assumptions.

The inductance of a line can be calculated if one knows the line length l, line geometry (i.e., physical spacing of the conductors) and the line material properties. With knowledge of the line reactance, We can approximate the line length using estimates of the line geometry and material properties. We could do something similar with the line capacitances and resistances, but these values may not always be specified in the data set, whereas reactance values should always be specified.

The formula for inductance is

$$L = 2 \times 10^{-7} \ln \left(\frac{D_{eq}}{D_{SL}} \right) l \quad [\mathrm{H}] \tag{7}$$

where D_{eq} is the geometric mean distance (GMD) for the line conductors, which is dependent on the line geometry, and D_{SL} is the geometric mean radius (GMR) for the line conductors, which is dependent on the conductor characteristics (stranding, conductor bundling, etc.). *Note that all quantities must be converted to SI units in this equation (meters and Henrys).*

The value of l can be directly calculated with knowledge of L, D_{eq} and D_{SL}. We next describe how to obtain these values. Table A.4 of [2] provides properties for a variety of conductor materials. Using the typical conductor sizes given in Figure 7 of [3] and the material properties from Table A.4 of [2], relevant values of the geometric mean radius for a single conductor (D_S) are shown in Table 3.

Conductor Name	Conductor Size (kcmil)	D_S (feet)
Drake	795	0.0375
Cardinal	954	0.0403

Table 3: Conductor Material Properties (Table A.4 in [2])

Accounting for the effects of conductor bundling, the value of D_{SL} is computed from the per-conductor GMR values (D_S). Assuming a symmetric arrangement of conductors within each bundle with a distance d between conductors that is much greater than the conductor radii (i.e., $d \gg D_S$), D_{SL} is calculated as

Two-Conductor Bundle	$\sqrt{D_S d}$
Three-Conductor Bundle	$\sqrt[3]{D_S d^2}$
Four-Conductor Bundle	$1.091 \sqrt[4]{D_S d^3}$

Table 4: Calculations for GMR (D_{SL}) with Conductor Bundling [2]

A typical distance d between conductors in a bundle is 18 inches [4].

The remaining value necessary for determination of the line length l is the geometric mean distance D_{eq} for the line. This value is a function of the interconductor distance D. For a completely transposed, horizontal line configuration where the interconductor distance is much greater than the bundle distance (i.e., $D \gg d$), D_{eq} is calculated as

$$D_{eq} = D\left(2^{\frac{1}{3}}\right) \tag{8}$$

Typical interconductor distances D vary with the voltage of the transmission line; higher voltage lines require larger interconductor distances for insulation purposes. Approximating the interconductor distance as a linear function of the voltage level, we use the linear interpolation in (9) to estimate the interconductor distance D for voltages that do not appear in Table 5. Enforce a minimum value of 1 foot for D.

$$D = 0.077\,(\text{Voltage in kV}) - 3.11 \qquad [\text{Feet}] \tag{9}$$

Voltage (kV)	Interconductor Distance D (feet)	Reference
115	3	[1]
345	25	[5]
500	40	[5]
735	50	[5]

Table 5: Interconductor Distance D for Various Voltages

With these assumptions and the value of L from the power flow data set, calculate the line length l using (7). Reasonable values of l are between tens to hundreds of miles.

3 Line-Flow Limit

A practical line-flow limit can now be calculated using the values for SIL and l. To account for short lines (less than 50 miles), enforce a maximum line-flow limit of $3.0 \times$ SIL. For longer lines, we use the line loadability characteristic described in Figure 7 of [3], which is reproduced in Figure 1.

A power function interpolation of the loadability characteristic gives the following relationship between loadability in SIL (S^{max}) and line length l.

$$S^{max} = \text{Loadability in multiples of SIL} = 42.40\,(\text{Length in Miles})^{-0.6595} \tag{10}$$

The value for S^{max} should be somewhere between $0.5 \times (SIL)$ (long lines) and $3 \times (SIL)$ (short lines).

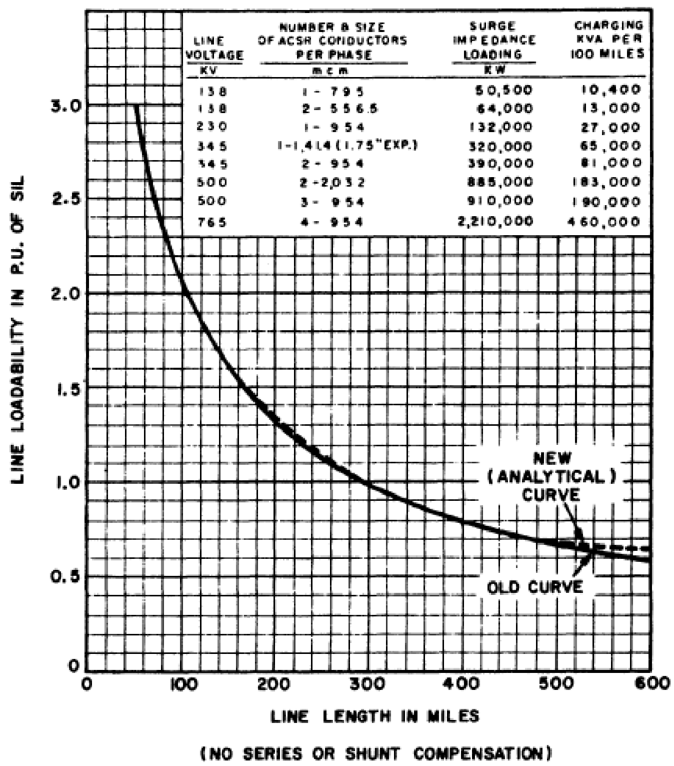

Figure 7. Comparison between "analytical" and "old" curves

Figure 1: Loadability Characteristic (Figure 7 of [3])

4 Current Flow Limits

The limits developed thus far are based on apparent power ("MVA") line flows. Limits based on the magnitudes of current flows are more naturally incorporated in formulations of the optimal power flow problem that use currents as optimization variables. We propose three line-flow limits based on current flows:

1. Conversion from original apparent power flow limits.
2. Conversion from estimated apparent power flow limits.
3. Estimation from typical line parameters.

Obtaining current-based flow limits from the original or estimated apparent power requires a straightforward conversion. First convert the line flow limits from MVA to per unit representation by dividing by the base power, which is specified to be 100 MVA. Using the approximation that per unit voltage magnitudes are 1.0 per unit at all buses, the per unit current limits are then equal to the per unit apparent power flow limits. (This approximation is also used in the current flow limit conversion in MATPOWER [6] with the option `OPF_FLOW_LIM` equal to 2.) The first two current flow limits apply this conversion to the apparent power limits specified in the original data set and the apparent power limits developed using the method described in this document.

The estimation of current flow limits based on typical line parameters uses the current limits specified for typical transmission lines. Therefore, many of the caveats for the apparent power approach detailed in Section 1 also apply to this approach. That is, this approach is only applicable for typical transmission lines; current limits cannot be developed in this manner for transformers and for lines that are part of equivalenced system models.

In this derivation, we use the number of conductors for each voltage level from Table 2. Conductor material properties are obtained from Table A.4 of [2]. The 795 kcmil conductors for 138 kV transmission lines use the material properties for the "Drake" conductor type and the 954 kcmil conductors for the remaining transmission voltages use the material properties for the "Cardinal" conductor type.

Table A.4 of [2] gives the approximate current carrying capacity in amps for one cable of these conductors. The current carrying capacity is based on a conductor operating temperature of 75°C with an ambient temperature of 25°C, wind at 1.4 miles per hour, and a system frequency of 60 Hz. The relevant data are reproduced as Table 6.

Conductor Name	Approx. Current Carrying Capacity (amps)
Drake	900
Cardinal	1010

Table 6: Approximate Current Carrying Capacity (Table A.4 in [2])

To derive per unit current flow limits from these data, first multiply the current carrying capacity from Table 6 by the number of bundled conductors for the appropriate voltage level specified in Table 2. This gives the maximum current flow per phase in amps. Next convert from amps to per unit representation. This requires the base current I_{base}.

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} \qquad \text{[amps]} \tag{11}$$

where S_{base} is the three-phase apparent power base and V_{base} is the line-to-line base voltage. These quantities are typically directly specified in power flow data sets. Note that I_{base} is in amps when S_{base} is in VA and V_{base} is in volts (e.g., $S_{base} = 100 \times 10^6$ VA and $V_{base} = 138 \times 10^3$ V).

The estimated current flow limits are then obtained by dividing the maximum current flow per phase by the base current I_{base}.

References

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