Estimating Generator Capability Curves

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Simplistic generator models often use "rectangle constraints" for active and reactive output limits. That is, generators' capabilities are modeled with independent maximum and minimum limits on active and reactive power output (i.e., reactive power output is not a function of active power output). Although the rectangle constraints model provides a good first approximation, more detailed modeling is necessary to accurately characterize generator capability curves, which are also called "D-curves."

Using machine rating standards and the rectangle constraints commonly specified in power system data sets, we develop approximations of the capability curves for typical generators. Note that this work does not yet include development of lower active power generation limits; future work will develop lower limits on active power generation from knowledge of the generator's prime mover type.

Also note that many power flow solution software packages (e.g., MATPOWER, Power-World, and PSS/E) use piecewise-linear curves to model generator capability curves. The circular curves defined in this document can be converted to piecewise-linear curves by sampling at a variety of points.

1 Typical Capability Curve

The reactive power output of a synchronous generator is constrained by several factors: armature current limit, field current limit, and end region heating limit [1]. Each of these limits are modeled as circles in the active-reactive power output plane. The machine must operate within the intersection of these circles. The generator must also operate within maximum and minimum active power limits imposed by the prime mover.

Fig. 1 from reference [2] (reproduced as Fig. 1 in this document) shows the capability curve for a typical synchronous generator. The upper portion of the curve is the circle from the field current limit, the right portion of the curve is the circle from the armature current limit, and the lower portion of the curve is the circle from the end region heating limit. Each of these limits are due to I^2R heating of the corresponding section of the synchronous generator.

We develop approximations for each of these circles using specified rectangle constraints and knowledge of typical intersection points from machine rating standards. The rectangle

constraints are denoted as P^{max} and P^{min} for limits on active power output and Q^{max} and Q^{min} for limits on reactive power output.

These approximations rely on the rectangle constraints representing a single round-rotor synchronous generator. This analysis is not applicable to rectangle constraints that represent aggregations of generators and related equipment (e.g., switched capacitors, load demand at the generator bus, etc.).
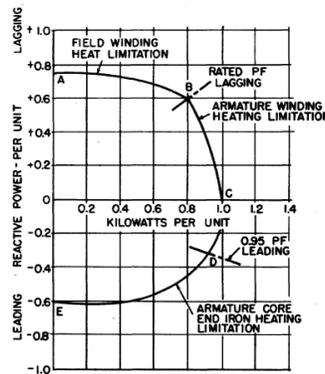
Figure 1: Typical Generator Capability Curve (Figure 1 in reference [2])

2   Minimum Power Factor

Minimum power factor is computed from the given generator minimum limits:

$$\text{min\_pf} = \frac{|P^{min}|}{\sqrt{(P^{min})^2 + (Q^{min})^2}} \tag{1}$$

3   Armature Current Limit

The armature current limit defines the right side of the capability curve. This limit is defined using the rated MVA of the generator. The armature current limit is described by a circle with center at the origin and radius equal to the rated MVA of the generator. Lacking more detailed information, we define

$$R^{max} = \max\left(P^{max}, Q^{max}\right) \tag{2}$$

as the R^{max}, maximum radius of the circle. The constraint for the armature current limit is

$$P^2 + Q^2 \leq \left(R^{max}\right)^2 \tag{3}$$

where P and Q denote the active and reactive power outputs of the generator.

A value of R^{max} that is less than P^{max} is interpreted as a maximum mechanical input power limit that is below the maximum electrical power generation limit of the synchronous generator. For such cases, a maximum active power limit of P^{max} must also be imposed on the capability curve.

4 Field Current Limit

The field current limit defines the top portion of the capability curve. An equation for this limit in terms of generator parameters is derived in [1]. However, typical power flow data sets do not have sufficient information for direct use of the equations from this derivation. Alternatively, we use knowledge of standard machine ratings to approximate this limit.

A circle has two degrees of freedom in the location of the center $\left(P_0^{field}, Q_0^{field}\right)$ and one degree of freedom in the radius $\left(r^{field}\right)$. We therefore need three pieces of information to define the field current limit.

We first use the fact that the field current limit is a circle with center $\left(P_0^{field}, Q_0^{field}\right)$ on the Q-axis; that is, $P_0^{field} = 0$ [1]. We then assume that the maximum reactive power output Q^{max} is achieved at zero active power output, which indicates that the point $(0, Q^{max})$ is on the circle. Finally, we use the fact that standard machine specifications use operation at rated power factor as the intersection between the field current limit and the armature current limit. Assuming a rated power factor of 0.80 lagging, this gives a second point on the circle: $(0.8R^{max}, 0.6R^{max})$. The parameters for the field current limit circle are then given by a solution to

$$\left(0.8R^{max}\right)^2 + \left(0.6R^{max} - Q_0^{field}\right)^2 = \left(r^{field}\right)^2 \tag{4a}$$

$$\left(Q^{max} - Q_0^{field}\right)^2 = \left(r^{field}\right)^2 \tag{4b}$$

Solving (4) yields

$$Q_0^{field} = \frac{\left(Q^{max}\right)^2 - \left(R^{max}\right)^2}{2\left(Q^{max} - 0.6R^{max}\right)} \tag{5a}$$

$$r^{field} = Q^{max} - Q_0^{field} \tag{5b}$$

The resulting field current limit is

$$P^2 + \left(Q - Q_0^{field}\right)^2 \le \left(r^{field}\right)^2 \tag{6}$$

The relative values of Q^{max} and P^{max} result in three cases of interest:

1. From (2), if $Q^{max} \ge P^{max}$, then $R^{max} = Q^{max}$ and $Q_0^{field} = 0$. For this case, the armature current and field current constraints are identical. Recall that a maximum active power limit (i.e., $P \le P^{max}$) is specified. A typical curve for this case is shown in Fig. 2.
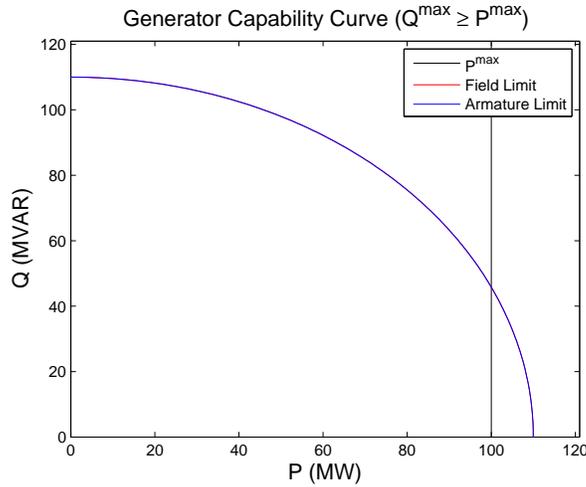
Figure 2: Generator Capability Curve $(Q^{max} \ge P^{max})$

2. If $Q^{max} \le 0.6P^{max}$, then Q_0^{field} is non-negative. Only negative values of Q_0^{field} are physically meaningful (see [1]). Accordingly, if $Q_0^{field} \ge 0$, we will choose to represent the upper portion of the capability curve as a horizontal line (i.e., $Q \le Q^{max}$) and disregard the value of Q_0^{field}. A typical curve for this case is shown in Fig. 3.

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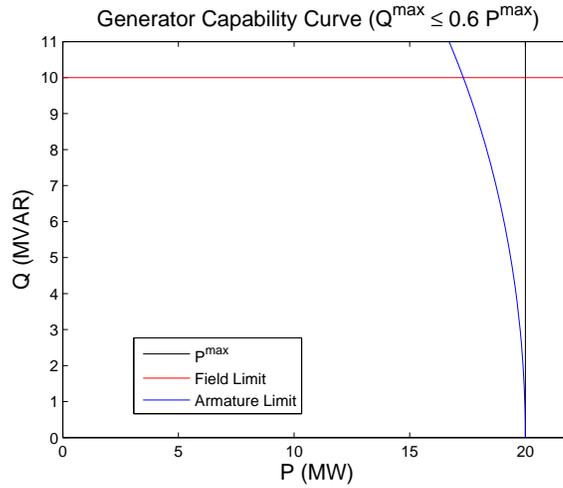
Figure 3: Generator Capability Curve ($Q^{max} \leq 0.6P^{max}$)

3. If $P^{max} \geq Q^{max} > 0.6P^{max}$, which is expected to be the case for typical generators, the armature and field current limits impose distinct circle constraints. A typical curve is shown in Fig. 4.
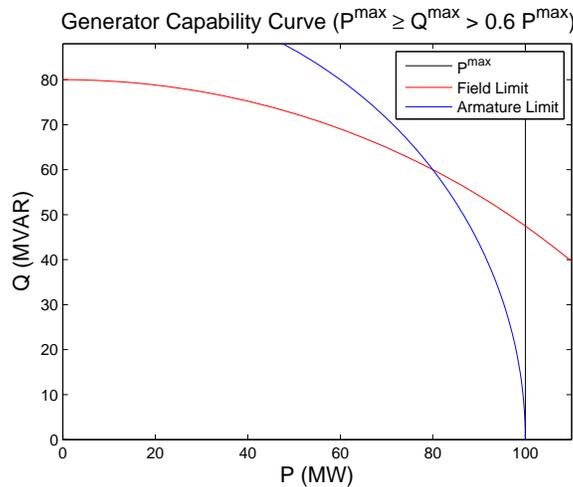
Figure 4: Generator Capability Curve ($P^{max} \geq Q^{max} > 0.6P^{max}$)

5 End Region Heating Limit

The lower limits of the capability curve are due to heating of the end regions of the synchronous generator. There is not a detailed derivation for these limits from the generator parameters as in the case of the field current limits [1].

We use several assumptions to approximate these limits in the same manner as the field current limits. Specifically, we assume 1) the end region heating limit takes the form a circle with center $\left(P_0^{end}, Q_0^{end}\right)$ on the Q-axis (i.e., $P_0^{end} = 0$) and radius r^{end}, 2) the point $(0, Q^{min})$ is on this circle, and 3) the intersection of this limit with the armature current limit occurs at 0.95 power factor leading as in Fig. 1.

Using these assumptions to follow a similar development as for the field current limits, we obtain

$$Q_0^{end} = \frac{(Q^{min})^2 - (R^{max})^2}{2\left(Q^{min} + 0.31 R^{max}\right)} \tag{7a}$$

$$r^{end} = Q_0^{end} - Q^{min} \tag{7b}$$

The resulting end region heating limit is

$$P^2 + \left(Q - Q_0^{end}\right)^2 \leq \left(r^{end}\right)^2 \tag{8}$$

The relative values of Q^{min} and R^{max} result in three cases of interest:

1. If $|Q^{min}| < 0.31 R^{max}$, then Q_0^{end} is non-positive. Only positive values of Q_0^{end} are physically meaningful (see [1]). Accordingly, if $Q_0^{end} \leq 0$, we will choose to represent the upper portion of the capability curve as a horizontal line (i.e., $Q \geq Q^{min}$) and disregard the value of Q_0^{end}. A typical curve for this case is shown in Fig. 5.
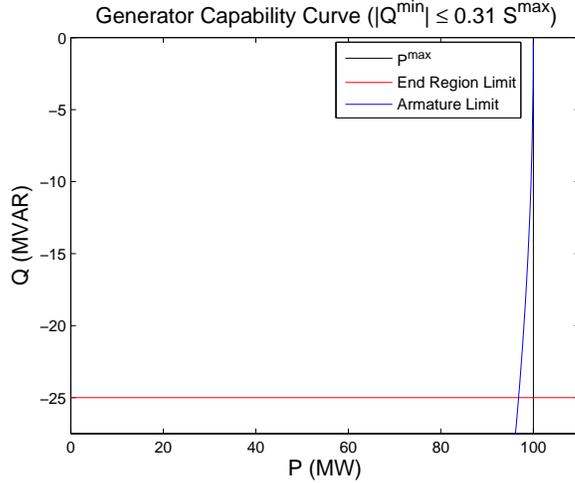
Figure 5: Generator Capability Curve ($|Q^{min}| \leq 0.31 R^{max}$)

2. If $S^{max} \geq |Q^{min}| > 0.31 R^{max}$, which is expected to be the case for typical generators, the armature and field current limits impose distinct circle constraints. A typical curve is shown in Fig. 6.

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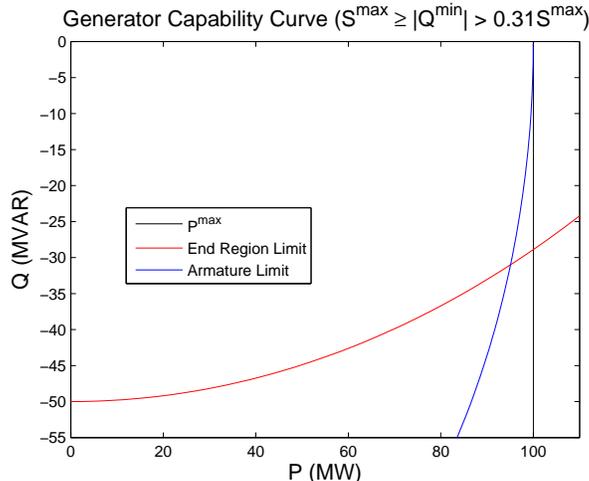
Figure 6: Generator Capability Curve ($R^{max} > |Q^{min}| > 0.31 R^{max}$)

3. The case $|Q^{min}| > R^{max}$ is atypical for synchronous generators. For this case, the armature current limit is binding before the specified Q^{min}; that is, the synchronous generator cannot actually reach Q^{min} due to the armature current limit. If this case does occur, use a horizontal line as the lower limit (i.e, $Q \geq Q^{min}$) and ignore the armature current limit for negative values of Q. In other words, use the original rectangle constraints for the lower half of the generator capability curve. Warn the user that an atypical case has occurred.

6   Prime Mover Limits

Limits on the mechanical input power from the prime mover impose constraints on the active power generation. The maximum and minimum active power generation P^{max} and P^{min} are determined directly from the rectangle constraints.

For typical coal, natural gas, oil, and wood fired generators, minimum active power generations that are set to zero (i.e., $P^{min} = 0$) likely indicate missing or incorrect data. A statistical study of the minimum economic operating point ("eco-min") of these generators is presented in [3]. This study uses the values of P^{min} from a dataset containing generators in PJM. To estimate P^{min}, we use the results of [3] corresponding to the data that is often available in power flow data sets.

Power flow data sets specify generators' nameplate capacity (i.e., P^{max}) and often provide generators' prime mover type (i.e., steam turbine, combustion turbine, combined cycle combustion turbine, etc.). If both of these data fields are available, we use the median eco-min data specified in Figures 15, 23, and 25 of [3] to approximate P^{min}. This data is reproduced in Table 1, which considers steam turbines and combined cycle prime movers, and Table 2, which considers combustion turbine prime movers operated both independently and as part of a combined cycle plant. Values of P^{min} are specified as a

percentage of P^{max}. Note that [3] indicates the possibility of substantial variance around these median data.

P^{max}	Steam Turbine	Combined Cycle
0-200 MW	38%	80%
200-400 MW	39%	46%
400-600 MW	49%	41%
600-800 MW	60%	48%
> 800 MW	64%	42%

Table 1: Typical P^{min} for Steam Turbines and Combined Cycle Prime Movers (Figures 15 and 23 in [3])

P^{max}	Independently Operated CT	CT in Combined Cycle Plant
0-50 MW	76%	80%
50-100 MW	66%	95%
100-150 MW	59%	63%
150-200 MW	81%	63%
200-250 MW	71%	58%
250-300 MW	–	64%

Table 2: Typical P^{min} for Combustion Turbine Prime Movers (Figure 25 in [3])

If the power flow data does not include the prime mover type, we use the averages among all prime mover types from Figure 10 of [3] (reproduced as Table 3) to specify P^{min} based on the nameplate capacity data only. Note that [3] indicates the possibility of substantial variance around these median data.

P^{max}	P^{min}
0-200 MW	69%
200-400 MW	42%
400-600 MW	45%
600-800 MW	48%
> 800 MW	69%

Table 3: Typical P^{min} Without Prime Mover Data (Figure 10 in [3])

7   Resulting Capability Curve

An example of the capability curve resulting from the procedure in this document is given in Fig. 7.
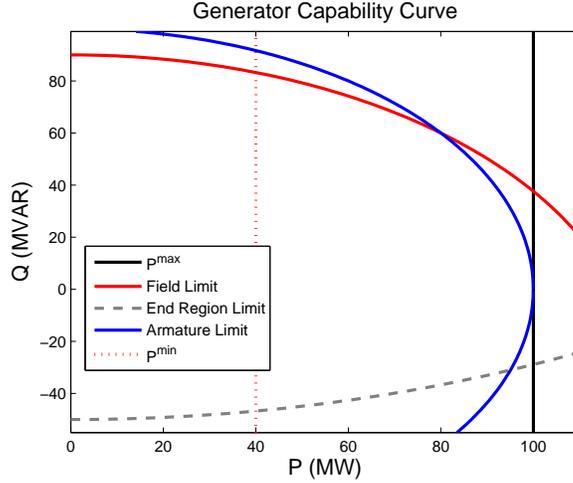
Figure 7: Generator Capability Curve

8 Limitation of the D-curve

A nonlinear generator D-curve constraint for use in AC models were imposed at all generator buses, but some polish cases produce infeasiblity since there are generators that contain no sufficient ratio of $\frac{P_{max}}{P_{min}}$ or $\frac{Q_{max}-Q_{min}}{Q_{max}}$. To cross off generators that are not suitable to impose the D-curve constraint, heuristic criterions for the ratio $\frac{P_{max}}{P_{min}}$ and $\frac{Q_{max}-Q_{min}}{Q_{max}}$ are added and the D-curve constraint is not applied for generators that have $\frac{P_{max}}{P_{min}} \leq 1.1$ or $\frac{Q_{max}-Q_{min}}{Q_{max}} \leq 0.1$. When this happens, it shows EPS in the relevant fields to indicate that the D-curve constraint will not be imposed on those generators. Figure below explains when to impose the D-curve constraint. The fiqure (a) shows the example of generator that as worst case produces a fixed output. So it is not applicable to impose the D-curve, whereas the figure (b) describes the generator that has resonable ratio of $\frac{P_{max}}{P_{min}}$ and $\frac{Q_{max}-Q_{min}}{Q_{max}}$ so that the D-curve constraint is implemented.

(a) Generator not suitable

(b) Generator suitable
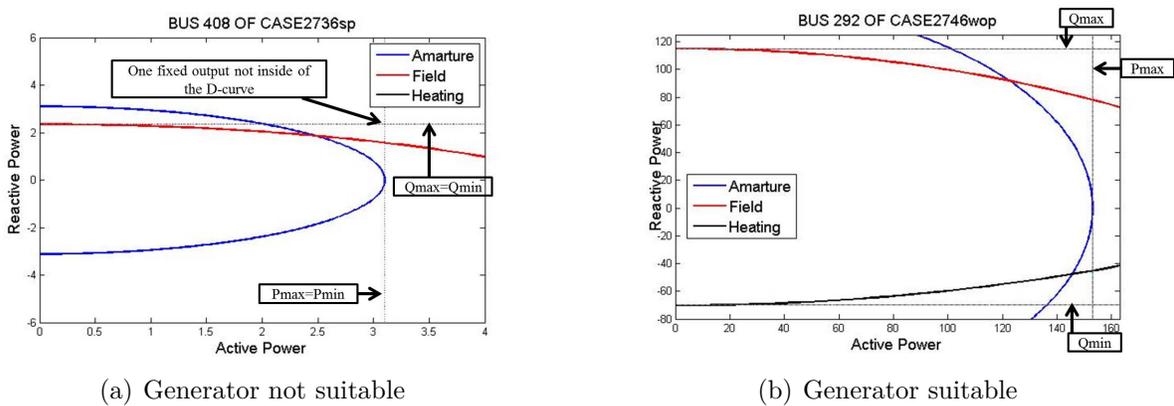
Figure 8: Example of when to impose the D-curve constraint

9 Nonphysical Data

Because these curves are computed from the given values of $P^{max}, P^{min}, Q^{max}$, and Q^{min}, in the case of atypical or unrealistic generator profiles (for example in the IEEE testcases) the computed constraints may be physically incorrect or meaningless. When this happens, the data reflects this by showing NA in the relevant fields so the modeler will know not to impose constraints based on improper values.

An additional "uwcalc" set of values are also provided that update the information so the all the profiles include tighter D-curve constraints than the original data generates.

References

[1] P. Kundur, N. Balu, and M. Lauby, *Power System Stability and Control.* McGraw-hill New York, 1994, vol. 4, no. 2.

[2] J. Jackson, "Interpretation and Use of Generator Reactive Capability Diagrams," *IEEE Transactions on Industry and General Applications*, no. 6, pp. 729–732, 1971.

[3] Federal Energy Regulatory Commission, "Eco-Min Estimation," Tech. Rep., 2013.