# Examination of ACOPF Solution Characteristic

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May 20, 2015

## 1 Examination Description

To evaluate ACOPF solution characteristic, three different ACOPF model formlations (polar power-voltage, rectangular power-voltage and rectangular current-voltage) are utilized. Each formulation has two versions to construct the probelm - one uses the addmittance matrix, which is known as $Y_{bus}$ and another use a summation of transmission line power flows to calculate power balance equations at every node. For this task, thermal limits are expressed in terms of real power, but current is used for the rectangular current-voltage formulation. Objective function of problems is to minimize quadratic generator operating cost curves. In addition, two cases - with/without the D-curve constraint are examined. The selection of three solvers (KNITRO, CONOPT, IPOPTH) is based on the expert experience that those are best suited for solving the ACOPF problems. The solver SNOPT is crossed off since it is not as effective as other solvers.

## 2 ACOPF Solution Characteristic

### 2.1 Notation

1. `ic=*`
   Initial condition for ACOPF problems and `*` refers to options to choose a different starting point.
2. `Ybus`
   ACOPF problems which use the admittance matrix to calculate node power balance equations.
3. `S.L.P`
   ACOPF problems which use the summation of line power flows to calculate node power balance equations.
4. `Time` : CPU time to solve ACOPF problems.
   - First : CPU time from generating starting points to a solution
   - Start : CPU time from starting points to a solution
   - Section 2.2 considers only `First` time to see general results and then section 2.3 considers both CPU time (`First and Start`) for more detail.
5. `O.V`
   Optimal objective value.

## 2.2 Optimal solution without the D-curve constraint

When it solves DCOPF problems for initial contions `ic=4,5`, CONOPT is used as a defualt solver. To solve the decoupled ACOPF for the initial contion `ic=6`, the same solver that ACOPF problems use is applied for consistency. For example, if KNITRO is used to examine ACOPF problem solutions, then the decoupled ACOPF problem which will be served as an initial condition also use KNITRO. Default values for each solver's option are used for end-users unfamiliar with GAMS.

| Solver : KNITRO | | POLAR | | | | RECTANGULAR | | | | IV-REC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | S.L.P | | Ybus | | S.L.P | | Ybus | | S.L.P | |
| | | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V |
| case14 | ic=1 | 0.5sec | 8081$ | 0.5sec | 8081$ | 0.55sec | 8081$ | 0.55sec | 8081$ | 0.7sec | 8081$ | 0.5sec | 8081$ |
| | ic=2 | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.5sec | 8081$ | 0.5sec | 8081$ |
| | ic=3 | infeas | infeas | infeas | infeas | 0.55sec | 8081$ | 0.55sec | 8081$ | infeas | infeas | infeas | infeas |
| | ic=4 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=5 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=6 | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ |
| case118 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 0.65sec | 129.66K$ | 0.6sec | 129.66K$ | 0.8sec | 129.66K$ | 0.7sec | 129.66K$ | 0.8sec | 129.66K$ | 0.8sec | 129.66K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 1.4sec | 129.66K$ | 1.4sec | 129.66K$ | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ | 1.6sec | 129.66K$ | 1.5sec | 129.66K$ |
| | ic=5 | 1.4sec | 129.66K$ | 1.4sec | 129.66K$ | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ |
| | ic=6 | 3sec | 129.66K$ | 3sec | 129.66K$ | 3sec | 129.66K$ | 3sec | 129.66K$ | 3.5sec | 129.66K$ | 3.5sec | 129.66K$ |
| case300 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 1sec | 719.72K$ | 1sec | 719.72K$ | 1.3sec | 719.72K$ | 1.2sec | 719.72K$ | 2.5sec | 719.72K$ | 1sec | 719.72K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 2sec | 719.72K$ | 2sec | 719.72K$ | 3sec | 719.72K$ | 2.5sec | 719.72K$ | 2.5sec | 719.72K$ | 2sec | 719.72K$ |
| | ic=5 | 2sec | 719.72K$ | 2sec | 719.72K$ | 2.3sec | 719.72K$ | 2.3sec | 719.72K$ | 2.5sec | 719.72K$ | 2sec | 719.72K$ |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| case2737sop | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 8sec | 777.624K$ | 7sec | 777.624K$ | 10sec | 777.624K$ | 10sec | 777.624K$ | 10sec | 777.624K$ | 7.5sec | 777.624K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 10sec | 777.624K$ | 9sec | 777.624K$ | 12.5sec | 777.624K$ | 11sec | 777.624K$ | 13.5sec | 777.624K$ | 9.5sec | 777.624K$ |
| | ic=5 | 10.5sec | 777.624K$ | 9.5sec | 777.624K$ | 13sec | 777.624K$ | 10.5sec | 777.624K$ | 12.5sec | 777.624K$ | 9.5sec | 777.624K$ |
| | ic=6 | 25.5sec | 777.624K$ | 23sec | 777.624K$ | 28sec | 777.624K$ | 26sec | 777.624K$ | 26.5sec | 777.624K$ | 24sec | 777.624K$ |
| case3120sp | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 10.5sec | 2.1422M$ | 8.5sec | 2.1422M$ | 15sec | 2.1422M$ | 11.5sec | 2.1422M$ | 15sec | 2.14117M$ | 9.5sec | 2.14117M$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 15sec | 2.1422M$ | 13sec | 2.1422M$ | 19sec | 2.1422M$ | 15sec | 2.1422M$ | 18sec | 2.14117M$ | 13sec | 2.14117M |
| | ic=5 | 15sec | 2.1422M$ | 13sec | 2.1422M$ | 19sec | 2.1422M$ | 15sec | 2.1422M$ | 20sec | 2.14117M$ | 13sec | 2.14117M |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |

Table 1: ACOPF solution characteristic with KNITRO solver

Notice that the yellow-colored backgrounds indicate formulation and initial condition that show best performance in terms of CPU time and robustness for the ACOPF problem. The red-colored backgrounds refer to an optimal solution which is a different local minimum from other solutions in the table.

- `Observation`
  - For large test cases, `S.L.P` version provides an optimal solution faster than `Ybus` version since redundant calculations for nodal balance equations are eliminated in `S.L.P` version. Also, they provide an equivalent optimal solution so task 3 and 5 only focus on `S.L.P` version.
  - It is ambiguous to make the decision which formulation is superior to other formulation with small test cases since their CPU time are all similar each other with every initial condition.
  - Infeasibility occurs only with random initial condition(ic=1&3) and decoupled ACOPF solution(ic=6). Since voltage angles are chosen randomly bewteen $-\pi$ and $\pi$ for ic=1&3 it could start at a strange point such that two adjacent buses have $-\pi$ and $\pi$ respectively in an extreme case so possibly create infeasiblity. For ic=6, it is due to the inherent shortcoming of the decoupled ACOPF[1], which creates infeasibility when the demand of the system is sufficiently large.
  - Polar power-voltage formulation with a summation of line power flows to calculate node balance equations seems most promising for the ACOPF.

[1]refer to the paper : ByungKwon Park and Christopher L. DeMarco,"Active/reactive power decomposition approaches to the AC optimal power flow problem,"North American Power Symposium (NAPS), 7-9 September 2014,

- Second initial condition, which is “Flat start”, where all initial guesses for voltage magnitude and angle are set to 1.0 and zero respectively, and power flow are set to zero, provides the best performance for the ACOPF starting point.
- Since the ACOPF is a nonlinear and nonconvex optimization problem, the global optimal solution is not guaranteed and only local solutions are obtained. Based on this fact, the rectangular current-voltage formulation obtains an optimal objective value lower than other formulations.

### 2.2.1 Comparison between the Polar and IV-REC formulation solution

Due to lack of more detailed information, the real power flow limit which are given us, as opposed to the current flow limit are used to impose the line limit for the rectangular current-voltage formulation. This leads us to formulate a slightly different model with the polar power-voltage and rectangular current-voltage formulation of the ACOPF problem. For this reason, the rectangular current-voltage formulation provides a different local minima for some cases.

Figure below shows differences between the polar power-voltage formulation and rectangular current-voltage formulation by calculaing the $l^2$-norm of optimal solution with two test cases; case 3120sp and case2737sop. The $l^2$-norm for each variable is equivalent when they produce the same optimal objective value(case2737sop), whereas if they obtain a different optimal objective value(case3120sp), then the small difference of the $l^2$-norm for each variable between the polar power-voltage formulation and rectangular current-voltage formulation is observed. This tells us that we find a different local solution.

**L-2 Norm of the optimal solution**

| | ***Case3120sop*** Produce a different optimal objective value | | ***Case2737sop*** Produce the same optimal objective value | |
|---|---|---|---|---|
| | Polar | IV-REC | Polar | IV-REC |
| Voltage Angle | 14.2015 | 14.4381 | 8.1233 | 8.1233 |
| Voltage Magnitude | 61.0426 | 61.1168 | 57.0679 | 57.0679 |
| Active Power Injection | 21.2463 | 21.2414 | 14.5362 | 14.5362 |
| Reactive Power Injection | 7.7249 | 7.8813 | 2.4746 | 2.4746 |

Figure 1: Comparison for different local minima

| Solver : CONOPT | | POLAR | | | | RECTANGULAR | | | | IV-REC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | S.L.P | | Ybus | | S.L.P | | Ybus | | S.L.P | |
| | | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V |
| case14 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.45sec | 8081$ | 0.4sec | 8081$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=5 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=6 | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ |
| case118 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ | 2sec | 129.66K$ | 2.5sec | 129.66K$ | 2sec | 129.66K$ | 1.7sec | 129.66K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 2sec | 129.66K$ | 2sec | 129.66K$ | 2.5sec | 129.66K$ | 2.5sec | 129.66K$ | 2.5sec | 129.66K$ | 2.5sec | 129.66K$ |
| | ic=5 | 2sec | 129.66K$ | 2.5sec | 129.66K$ | 2sec | 129.66K$ | 2.5sec | 129.66K$ | 2.2sec | 129.66K$ | 2.5sec | 129.66K$ |
| | ic=6 | 3.5sec | 129.66K$ | 3.5sec | 129.66K$ | 3.5sec | 129.66K$ | 4.5sec | 129.66K$ | 3.7sec | 129.66K$ | 4sec | 129.66K$ |
| case300 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 5sec | 719.72K$ | 10sec | 719.72K$ | 10sec | 719.72K$ | 17.5sec | 719.72K$ | 10sec | 719.72K$ | 11sec | 719.72K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 6sec | 719.72K$ | 7.5sec | 719.72K$ | 6.5sec | 719.72K$ | 11sec | 719.72K$ | 7.5sec | 719.72K$ | 6.5sec | 719.72K$ |
| | ic=5 | 4sec | 719.72K$ | 8sec | 719.72K$ | 5.5sec | 719.72K$ | 7sec | 719.72K$ | 6sec | 719.72K$ | 7sec | 719.72K$ |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| case2737sop | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 22sec | 777.624K$ | 23sec | 777.624K$ | 29sec | 777.624K$ | 37sec | 777.624K$ | 27sec | 777.624K$ | 31sec | 777.624K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 15sec | 777.624K$ | 32sec | 777.624K$ | 28sec | 777.624K$ | 42sec | 777.624K$ | 33sec | 777.624K$ | 54sec | 777.624K$ |
| | ic=5 | 2min33sec | 777.624K$ | 44sec | 777.624K$ | 50sec | 777.624K$ | 32sec | 777.624K$ | 33sec | 777.624K$ | 26sec | 777.624K$ |
| | ic=6 | 38sec | 777.624K$ | 52sec | 777.624K$ | 34sec | 777.624K$ | 40sec | 777.624K$ | 41sec | 777.624K$ | 34sec | 777.624K$ |
| case3120sp | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 3min10sec | 2.1422M$ | 45sec | 2.1422M$ | 3min33sec | 2.1422M$ | 1min23sec | 2.1422M$ | 6min | 2.14117M$ | 1min | 2.1405M$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 40sec | 2.1422M$ | 45sec | 2.1422M$ | 40sec | 2.1422M$ | 40sec | 2.1422M$ | 1min53sec | 2.14117M$ | 48sec | 2.1405M$ |
| | ic=5 | 45sec | 2.1422M$ | infeas | infeas | 40sec | 2.1422M$ | 56sec | 2.1422M$ | 2min | 2.14117M$ | 1min20sec | 2.1405M$ |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |

Table 2: ACOPF solution characteristic with CONOPT solver

| Solver : IPOPTH | | POLAR | | | | RECTANGULAR | | | | IV-REC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | S.L.P | | Ybus | | S.L.P | | Ybus | | S.L.P | |
| | | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V | Time | O.V |
| case14 | ic=1 | infeas | infeas | infeas | infeas | 0.6sec | 8081$ | 0.5sec | 8081$ | 0.5sec | 8081$ | infeas | infeas |
| | ic=2 | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.4sec | 8081$ | 0.45sec | 8081$ | 0.4sec | 8081$ |
| | ic=3 | infeas | infeas | infeas | infeas | 0.7sec | 8081$ | 0.6sec | 8081$ | 0.6 | 8081$ | 0.7sec | 8081$ |
| | ic=4 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=5 | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ | 1sec | 8081$ |
| | ic=6 | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ | 2sec | 8081$ |
| case118 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 0.8sec | 129.66K$ | 0.8sec | 129.66K$ | 1sec | 129.66K$ | 1sec | 129.66K$ | 0.9sec | 129.66K$ | 0.8sec | 129.66K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 1.5sec | 129.66K$ | 1.6sec | 129.66K$ | 1.5sec | 129.66K$ | 1.7sec | 129.66K$ | 1.7sec | 129.66K$ | 1.5sec | 129.66K$ |
| | ic=5 | 1.5sec | 129.66K$ | 1.5sec | 129.66K$ | 1.7sec | 129.66K$ | 1.5sec | 129.66K$ | 1.7sec | 129.66K$ | 1.5sec | 129.66K$ |
| | ic=6 | 3.3sec | 129.66K$ | 3sec | 129.66K$ | 3.2sec | 129.66K$ | 3sec | 129.66K$ | 3.5sec | 129.66K$ | 3sec | 129.66K$ |
| case300 | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 1sec | 719.72K$ | 1sec | 719.72K$ | 1.7sec | 719.72K$ | 1.5sec | 719.72K$ | 1.8sec | 719.72K$ | 1.2sec | 719.72K$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 2.2sec | 719.72K$ | 2sec | 719.72K$ | 2.5sec | 719.72K$ | 2.5sec | 719.72K$ | 2.5sec | 719.72K$ | 2.1sec | 719.72K$ |
| | ic=5 | 2sec | 719.72K$ | 2sec | 719.72K$ | 2.25sec | 719.72K$ | 2.2sec | 719.72K$ | 2.5sec | 719.72K$ | 2.1sec | 719.72K$ |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| case2737sop | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 7.5sec | 777.624K | 7sec | 777.624K | 11sec | 777.624K | 9sec | 777.624K | 10sec | 777.624K | 7sec | 777.624K |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 9sec | 777.624K | 8.5sec | 777.624K | 13sec | 777.624K | 10sec | 777.624K | 13.5sec | 777.624K | 9sec | 777.624K |
| | ic=5 | 9.5sec | 777.624K | 8.5sec | 777.624K | 13sec | 777.624K | 10sec | 777.624K | 12.5sec | 777.624K | 9.5sec | 777.624K |
| | ic=6 | 24sec | 777.624K | 23sec | 777.624K | 26sec | 777.624K | 24.5sec | 777.624K | 28sec | 777.624K | 23sec | 777.624K |
| case3120sp | ic=1 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=2 | 11sec | 2.1422M$ | 11sec | 2.1422M$ | 15sec | 2.1422M$ | 11sec | 2.1422M$ | 16sec | 2.14117M$ | 10sec | 2.1405M$ |
| | ic=3 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | 14sec | 2.1422M$ | 12sec | 2.1422M$ | 18.5sec | 2.1422M$ | 14sec | 2.1422M$ | 19sec | 2.14117M$ | 14sec | 2.1405M$ |
| | ic=5 | 13sec | 2.1422M$ | 14sec | 2.1422M$ | 20sec | 2.1422M$ | 16sec | 2.1422M$ | 2min | 2.14117M$ | 13sec | 2.1405M$ |
| | ic=6 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |

Table 3: ACOPF solution characteristic with IPOPTH solver

- `Observation`
  - Based on the Table 1,2 and 3 KNITRO and IPOPTH are better suited for solving the ACOPF problem than CONOPT.

– Polar power-voltage formulation provides best performance in terms of CPU time and rectangular current-voltage formulation obtains the same local solution achieving the lower optimal objective value in solvers, CONOPT and IPOPTH, as well.

– There are no signficant differences for the behavior of the ACOPF solution with small test cases. Infeasiblity happens to occur with only same initial conditions without alternating among other initial conditions. Thus next section for the examination including the D-curve constraint is couducted with only largest test cases, case2737sop and case3120sp and initial conditions(ic=1,3,6) that create infeasibility are not considered.

## 2.3 Optimal solution with the D-curve constraint

This section conducts the examination with the nonlinear generator D-curve constraint. All generator is applied to the D-curve constraint unless they have a failry small rectangular limit formed by $P_{max}\&P_{min}$ and $Q_{max}\&Q_{min}$. Heuristic criterions to cross off generators are implemented so that the D-curve is not applied to generators that have $\frac{P_{max}}{P_{min}} \leq 1.1$ or $\frac{Q_{max}-Q_{min}}{Q_{max}} \leq 0.1$.

| Solver : KNITRO | | POLAR | | | | | | RECTANGULAR | | | | | | IV-REC | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | |
| | | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V |
| | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | |
| case2737sop | ic=2 | 8sec | 8sec | 777.624K$ | 7sec | 7sec | 777.624K$ | 11sec | 11sec | 777.624K$ | 8sec | 8sec | 777.624K$ | 10.5sec | 10.5sec | 777.624K$ | 7.5sec | 7.5sec | 777.624K$ |
| | ic=4 | 10.5sec | 8.5sec | 777.624K$ | 10sec | 8sec | 777.624K$ | 14sec | 11sec | 777.624K$ | 10sec | 8sec | 777.624K$ | 13.5sec | 11.5sec | 777.624K$ | 11sec | 9sec | 777.624K$ |
| | ic=5 | 10sec | 8sec | 777.624K$ | 9.5sec | 7.5sec | 777.624K$ | 13sec | 11sec | 777.624K$ | 10.5sec | 8.5sec | 777.624K$ | 13.5sec | 11.5sec | 777.624K$ | 10sec | 8sec | 777.624K$ |
| | ic=6 | 35sec | 10sec | 777.624K$ | 44sec | 9sec | 777.624K$ | 47sec | 12sec | 777.624K$ | 45sec | 10sec | 777.624K$ | 46.5sec | 11.5sec | 777.624K$ | 45sec | 9sec | 777.624K$ |
| case3120sp | ic=2 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=5 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |

Table 4: ACOPF solution characteristic including the D-curve with KNITRO solver

| Solver : CONOPT | | POLAR | | | | | | RECTANGULAR | | | | | | IV-REC | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | |
| | | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V |
| | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | |
| case2737sop | ic=2 | 23sec | 23sec | 777.624K$ | 37sec | 37sec | 777.624K$ | 32sec | 32sec | 777.624K$ | 27sec | 27sec | 777.624K$ | 26sec | 26sec | 777.624K$ | 24sec | 24sec | 777.624K$ |
| | ic=4 | 15sec | 13sec | 777.624K$ | 28sec | 26sec | 777.624K$ | 26sec | 24sec | 777.624K$ | 4min40sec | 4min38sec | 777.624K$ | 21sec | 19sec | 777.624K$ | 20sec | 18sec | 777.624K$ |
| | ic=5 | 28sec | 26sec | 777.624K$ | 23sec | 21sec | 777.624K$ | 32sec | 30sec | 777.624K$ | 29sec | 27sec | 777.624K$ | 38sec | 36sec | 777.624K$ | 26sec | 24sec | 777.624K$ |
| | ic=6 | 37sec | 17sec | 777.624K$ | 44sec | 24sec | 777.624K$ | 45sec | 24sec | 777.624K$ | 44sec | 24sec | 777.624K$ | 39sec | 19sec | 777.624K$ | 33sec | 13sec | 777.624K$ |
| case3120sp | ic=2 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=4 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |
| | ic=5 | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas | infeas |

Table 5: ACOPF solution characteristic including the D-curve with CONOPT solver

| Solver : IPOPTH | | POLAR | | | | | | RECTANGULAR | | | | | | IV-REC | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | | Ybus | | | S.L.P | | |
| | | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V | Time | | O.V |
| | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | | First | Start | |
| case2737sop | ic=2 | 8sec | 8sec | 777.624K$ | 10sec | 10sec | 777.624K$ | 11sec | 11sec | 777.624K$ | 11sec | 11sec | 777.624K$ | 11sec | 11sec | 777.624K$ | 8sec | 8sec | 777.624K$ |
| | ic=4 | 9sec | 7sec | 777.624K$ | 10sec | 8sec | 777.624K$ | 12sec | 10sec | 777.624K$ | 13sec | 11sec | 777.624K$ | 14sec | 12sec | 777.624K$ | 10sec | 8sec | 777.624K$ |
| | ic=5 | 9.5sec | 7.5sec | 777.624K$ | 10sec | 8sec | 777.624K$ | 12.5sec | 10.5sec | 777.624K$ | 11sec | 9sec | 777.624K$ | 13.5sec | 11.5sec | 777.624K$ | 10sec | 8sec | 777.624K$ |
| | ic=6 | 26sec | 7sec | 777.624K$ | 25sec | 6sec | 777.624K$ | 33sec | 11sec | 777.624K$ | 31sec | 9sec | 777.624K$ | 29sec | 9sec | 777.624K$ | 27sec | 7sec | 777.624K$ |
| case3120sp | ic=2 | 10.5sec | 10.5sec | 2.1425M$ | 10sec | 10sec | 2.1425M$ | 20sec | 20sec | 2.1425M$ | 12.5sec | 12.5sec | 2.1425M$ | 18sec | 18sec | 2.1414M$ | 10sec | 10sec | 2.1408M$ |
| | ic=4 | 14sec | 10.5sec | 2.1425M$ | 13sec | 8sec | 2.1425M$ | 19sec | 15.5sec | 2.1425M$ | 17.5sec | 14sec | 2.1425M$ | 33sec | 30sec | 2.1414M$ | 16sec | 12.5sec | 2.1408M$ |
| | ic=5 | 14sec | 10.5sec | 2.1425M$ | 13sec | 8sec | 2.1425M$ | 19sec | 15.5sec | 2.1425M$ | 15sec | 11.5sec | 2.1425M$ | 50sec | 47sec | 2.1414M$ | 14.5sec | 11sec | 2.1408M$ |

Table 6: ACOPF solution characteristic including the D-curve with IPOPTH solver

- `Observation`
  - It is checked that all test case with the D-curve constraint works properly, which implies it does not create infeasibility. The CPU time to obtain an optimal solution is increased slightly for almost all case due to the D-curve constraint.
  - By imposing the D-curve constraint, we could recognize that an optimal objective value for some test cases are increased slightly. This is since the original rectangular generator limit is replaced with the D-curve, which is smaller than the original rectangular generator limit in size. This leads us to the reduction of our original feasible region.
  - IPOPTH solver shows better performance for the ACOPF problems than CONOPT and KNITRO in every portion. IPOPTH is faster and more robust, whereas CONOPT takes longer time to find an optimal solution. Morevoer, CONOPT and KNITRO fail to find feasible points for some cases with the D-curve constraint.
  - Decoupled ACOPF as an initial condition first seems to be an unattractive choice to start the ACOPF in terms of CPU time, which is `First` calculating time from generating initial guesses to an optimal solution. However, in terms of `Start` calculating time from starting points to an optimal solution, the decoupled ACOPF serves as a great initial condition.
  - In most cases, the polar power-voltage formulation with S.L.P version converges to an optimal solution faster than any other formulation. However, as discussed above the rectangular current-voltage formulation provides a different local minimum for some cases and converging time is slower than the polar power-voltage formulatin slightly.

# 3 Conclusion

- `Model`
  - Polar power-voltage formulation with the S.L.P version shows the best performance in terms of the convergent speed.
  - Rectangular current-voltage formulation tends to find a different local solution for some cases, but converges to an optimal solution slightly slower than the polar power-voltage formulation.
- `Initial guess`
  - Flat start provides the fastest convergence for the ACOPF when we calculate the CPU time from generating those initial guesses to an optimal solution.

  - Decoupled ACOPF provides the fastest convergence for the ACOPF with case2737sop when we calculate the CPU time from those initial guesses to an optimal solution. However, there exists the drawback that it happens to fail to find an initial points for some cases.

- `Solver`

  - IPOPTH serves as the most promising solver for ACOPF problems.
  - CONOPT and KNITRO has difficulties to find a feasible point for some cases when the D-curve constraint is applied to ACOPF problems.